Phrases as algebraic expressions

ALGEBRA BASICS
Phrases as Algebraic Expressions
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1 Write these phrases as algebraic expressions (let the number be 'n')

a The sum of a number and 7:

$n + 7$

b The difference between 9 and a number:

$n - 9$

c The sum of 6 times a number and 1:

$6n + 1$

d The product (×) of a number and 4:

$4n$

e The quotient (÷) of two more than a number and 3:

$\frac{n+2}{3}$

f The difference between a number squared and 6:

$n^2 - 6$

g The product of a number minus 5 and 2:

$2(n - 5)$

h 8 less than twice a number:

$2n - 8$

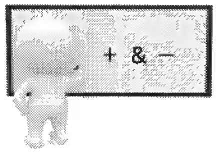
i 10 added to a number halved:

$\frac{n}{2} + 10$

j A number multiplied by 5 more than itself:

$n(n + 5)$

Addition and subtraction



If the variable parts are **exactly** the same, the terms are called 'like terms'.

Like terms: x and $-x$ (Like terms) $3b$ and b (Like terms) $2y$ and $-5y$ (Like terms)

Not Like terms: a and b (Not like terms) p and p^2 (Not like terms) $2x$ and $-5y$ (Not like terms)

Only 'like terms' can be added or subtracted.

Simplify $2a + a$

$$2a + a$$

Variable parts are the same (like terms)

$$\therefore 2a + a = 3a$$

Simplify $8x - 3x$

$$8x - 3x$$

like terms

$$\therefore 8x - 3x = 5x$$

Simplify $3d + 4d + 6c$

$$3d + 4d + 6c$$

like terms

$$3d + 4d + 6c = 7d + 6c$$

This cannot be simplified any further

Why don't we add or subtract unlike terms? Good Question!

Let's look at a problem the last example could represent.

At a picnic for pets, each dog gets 7 treats and each cat gets 6 treats. Number of treats needed is: (7 treats × number of dogs) + (6 treats × number of cats)

$$= (7 \times d) + (6 \times c)$$

the number of dogs the number of cats

Simplified: $= 7d + 6c$

d and c represent two different animals so it does not make sense to add them together.

Therefore $7d + 6c$ is the **simplest expression** for this problem.

Expanding

Grouping symbols such as [brackets], {braces} and (parentheses), can be removed without changing the value of the expression by expanding.

$$\begin{aligned} a(b+c) &= a\times b \text{ and } a\times +c \\ &= ab+ac \end{aligned}$$

Every term inside the parentheses is multiplied by the term in front of the parentheses.

Expand $2(3x+5)$

$$\begin{aligned} 2(3x+5) &= 2(3x+5) && \text{2} \times \text{every term inside the parentheses} \\ &= 2\times 3x \text{ and } 2\times +5 \\ &= 6x \text{ and } +10 \\ &= 6x+10 \end{aligned}$$

$$\begin{aligned} a(b+c) &= a\times b \text{ and } a\times -c \\ &= ab-ac \end{aligned}$$

Expand $3a(a-4)$

$$\begin{aligned} 3a(a-4) &= 3a(a-4) && 3a \times \text{every term inside the parentheses} \\ &= 3a\times a \text{ and } 3a\times -4 \\ &= 3a^2 \text{ and } -12a \\ &= 3a^2-12a \end{aligned}$$

Now that you have seen how it works, it's time to learn the mathematical name we give this is method:

The Distributive Law

- $$\begin{aligned} a(b+c) &= a\times b \text{ and } a\times +c \\ &= ab+ac \end{aligned}$$
- $$\begin{aligned} a(b-c) &= a\times b \text{ and } a\times -c \\ &= ab-ac \end{aligned}$$

Expanding

1 Expand:

a $2(a+7)$

$= 2a + 14$

b $9(b-3)$

$= 9b - 27$

c $6c(3d+1)$

$= 18cd + 6c$

d $4d(3-c)$

$= 12d - 4cd$

e $3x(6+4y)$

$= 18x + 12xy$

f $3m(p-q)$

$= 3mp - 3mq$

g $\frac{1}{2}(6m-14)$

$= 3m - 7$

h $2ab(3c+2d)$

$= 6abc + 4abd$

i $4(-3-9x)$

$= -12 - 36x$

j $-2p\left(2-\frac{q}{2}\right)$

$= -4p + pq$

More expanding

Why limit yourself to parentheses with only two terms? The Distributive Law works for parentheses with more.

Every term inside the parentheses is multiplied by the term in front.

Expand $4(2m+3n-2)$

$$\begin{aligned} 4(2m+3n-2) &= 4(2m+3n-2) && 4\times \text{ every term inside the parentheses} \\ &= 4\times 2m \text{ and } 4\times +3n \text{ and } 4\times -2 \\ &= 8m \text{ and } +12n \text{ and } -8 \\ &= 8m+12n-8 \end{aligned}$$

Take care with the multiplications when there is a negative term out the front.

Expand $-a(a-b+3c+2)$

$$\begin{aligned} -a(a-b+3c+2) &= -a(a-b+3c+2) && a\times \text{ every term inside the parentheses} \\ &= -a\times a \text{ and } -a\times -b \text{ and } -a\times +3c \text{ and } -a\times +2 \\ &= -a^2 \text{ and } +ab \text{ and } -3ac \text{ and } -2a \\ &= -a^2+ab-3ac-2a \end{aligned}$$

The basic index laws are often used when expanding expressions.

Expand $p^2(p-3pq+5q)$

$$\begin{aligned} p^2(p-3pq+5q) &= p^2(p-3pq+5q) && p^2\times \text{ every term inside the parentheses} \\ &= p^2\times p \text{ and } p^2\times -3pq \text{ and } p^2\times +5q \\ &= p^{2+1} \text{ and } -3p^{2+1}q \text{ and } +5p^2q \\ &= p^3-3p^3q+5p^2q \end{aligned}$$

Remember:
$a^m\times a^n = a^{m+n}$

More expanding

1 Expand:

a) $3(a+b+2)$

$= 3a + 3b + 6$

b) $4(x-y-5)$

$= 4x - 4y - 20$

c) $3p(2p+q+4)$

$= 6p^2 + 3pq + 12p$

d) $-d(e+2f+6)$

$= -de - 2df - 6d$

e) $2x(4x+3y-3+z)$

$= 8x^2 + 6xy - 6x + 2xz$

f) $-a(b-2c+d-5)$

$= -ab + 2ac - ad + 5a$

2 Expand: (psst: remember the multiplication rule for indices)

a) $n(n^2+3n)$

$= n^3 + 3n^2$

b) $xy(x^2-y^3)$

$= x^3y - xy^4$

c) $-ab(ab^2+2a^2b)$

$= -a^2b^3 - 2a^3b^2$

d) $2p(2p^2-4pq+5)$

$= 4p^3 - 8p^2q + 10p$

Expanding and simplifying

Always simplify the expression after expanding where possible.

Simplify by collecting like terms after the expansion of any parentheses.

Expand and simplify: $3(7m-6)-16m$

$$3(7m-6)-16m = 3(7m-6)-16m$$ 3 × every term inside the parentheses

$$= 3\times 7m \text{ and } 3\times -6 \text{ and } -16m$$

$$= \underbrace{21m - 18 - 16m}_{\text{Like terms}}$$

$$= 5m-18$$ Combine the like terms

For expressions with multiple parentheses, expand each separately then look to simplify.

Expand and simplify: $5(2a+4)-4(a-3)$

$$5(2a+4)-4(a-3) = 5(2a+4)-4(a-3)$$ Expand each grouping separately

$$5\times 2a \text{ and } 5\times +4 \qquad -4\times a \text{ and } -4\times -3$$

Like terms

$$= 10a+20-4a+12$$ Identify the like terms

Like terms

$$= 10a-4a+20+12$$ Group the like terms

$$= 6a+32$$ Simplify

Be careful to apply the index laws correctly when expanding expressions with multiple variables.

Expand and simplify: $xy(5x+y)-2x^2y$

$$xy(5x+y)-2x^2y = xy(5x+y)-2x^2y$$ xy × every term inside the parentheses

$$= xy\times 5x \text{ and } xy\times +y \text{ and } -2x^2y$$

$$= 5x^{1+1}y \text{ and } xy^{1+1} \text{ and } -2x^2y$$ Identify the like terms

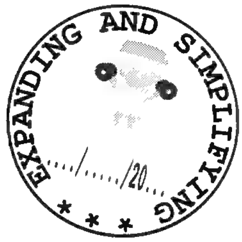
$$= 5x^2y + xy^2 - 2x^2y$$

Like terms

$$= 5x^2y - 2x^2y + xy^2$$ Group the like terms

$$= 3x^2y + xy^2$$ Simplify

Expanding and simplifying



1 Expand and simplify:

a $4(a+3)+2a$

$= 4a + 12 + 2a$

$= 6a + 12$

b $-3(2-x)+1$

$= -6 + 3x + 1$

$= -5 + 3x$

c $12p+5(p-2)$

$= 12p + 5p - 10$

$= 17p - 10$

d $5d-4(9-3d)$

$= 5d - 36 + 12d$

$= 17d - 36$

e $-5b(4-b)+3b+5b^2$

$= -20b + 5b^2 + 3b + 5b^2$

$= -17b + 10b^2$

f $9(x-2y)-x+4y$

$= 9x - 18y - x + 4y$

$= 8x - 14y$

2 Expand and simplify:

a $8(c-4)+3(c+2)$

$= 8c - 32 + 3c + 6$

$= 11c - 26$

b $9(d+2)-(5-3d)$

$= 9d + 18 - 5 + 3d$

$= 12d + 13$

c $3(x-5)-2(4+x)$

$= 3x - 15 - 8 - 2x$

$= x - 23$

d $a(a+8)-5(a+3)$

$= a^2 + 8a - 5a - 15$

$= a^2 + 3a - 15$

Expanding and simplifying

3 Expand and simplify:

a $-(y+4x)-5(2x-y)$

Psst! Remember the 1 can be hidden: $-1(y+4x)$

$= -y - 4x - 10x + 5y$

$= 4y - 14x$

b $x(2+x-y)+3x-xy$

$= 2x + x^2 - xy + 3x - xy$

$= 5x + x^2 - 2xy$

c $2a(3+4b)+4(ab+2a)$

$= 6a + 8ab + 4ab + 8a$

$= 14a + 12ab$

d $-3b(2+b)-(6-b)$

$= -6b - 3b^2 - 6 + b$

$= -5b - 3b^2 - 6$

e $-(2-d)-2(d-2)$

$= -2 + d - 2d + 4$

$= 2 - d$

f $xy(40x+5)-3y(10x^2-x)$

$= 40x^2y + 5xy - 30x^2y + 3xy$

$10x^2y + 8xy$

g $-mn(5m-2n^2)+mn^3+3m^2n$

$= -5m^2n + 2mn^3 + mn^3 + 3m^2n$

$= -2m^2n + 3mn^3$

h $q(4p+3q^2-2)+2q(q+5p)$

$= 4pq + 3q^3 - 2q + 2q^2 + 10pq$

$= 14pq + 3q^3 - 2q + 2q^2$

What is a Linear Equation?

An equation is a mathematical expression that has two sides separated by an equals sign (=) and at least one variable (or 'pronumeral'). If the highest power of the variable is 1 then the equation is called a 'linear equation'.

For example, these are all equations because '=' appears in each of them.

a $x + 4 = 12$ b $3x = 15$ c $2 + 3x = 8$ d $\frac{x}{3} = 5$ e $\frac{6x}{7} = 12$

These are also all **linear** because the power of x (the variable) is 1 (there is no x^2 or x^3 etc).

How are Equations Solved?

To solve ANY equation the goal is always to **get the variable by itself.** But whatever is done to find the variable by itself, must also be done to the other side. Each linear equation has only **one solution!**

Finding x by itself

If you look at the linear equation $x + 4 = 12$, it's probably easy to see $x = 8$. Your brain has actually simplified the equation to have x by itself without you even realising it.

This is how:

Subtracting 4 from the left side of the equation leaves x by itself. But the same must be done on the right side.

$$x + 4 = 12$$
$$x + 4 - 4 = 12 - 4$$
$$x = 8$$

For the equation in b, to find x by itself, both sides must be divided by 3

We must do the same to both sides

$$3x = 15$$
$$\frac{3x}{3} = \frac{15}{3}$$
$$x = 5$$

For the equation in c, it takes two steps to get x by itself

Subtract 2 from **both sides** to leave the term with x by itself

Divide **both sides** by 3 to leave the x by itself

$$2 + 3x = 8$$
$$2 + 3x - 2 = 8 - 2$$
$$3x = 6$$
$$\frac{3x}{3} = \frac{6}{3}$$
$$x = 2$$

If the variable is in the numerator of a fraction, like in d, then both sides must be multiplied by the denominator.

For the equation in d, we need to multiply the left hand side by 3 to find x by itself

$$\frac{x}{3} \times 3 = 5 \times 3$$
$$x = 15$$

Sometimes the variable will be in a fraction. Multiply and divide both sides to get the variable by itself.

In the equation in e x is the numerator of a fraction

Multiply **both sides** by the denominator (always remove the denominator first!)

$$\frac{6x}{7} \times 7 = 12 \times 7$$
$$6x = 84$$

Divide both sides by 6 to leave the x by itself

$$\frac{6x}{6} = \frac{84}{6}$$
$$x = 14$$

Sometimes the left side will have to be simplified by collecting like terms:

Solve for x in the following linear equation

$$2x + 5 + 3x = 30$$

Simplify by collecting like terms

$$5x + 5 = 30$$
$$5x + 5 - 5 = 30 - 5$$
$$5x = 25$$

Divide both sides by 5 to leave x by itself

$$\frac{5x}{5} = \frac{25}{5}$$
$$x = 5$$

What happens if the Power of the Variable is 2?

If the highest power of the variable is 2 (ie. x^2 is in the equation), then the equation is not linear, but is called a **Quadratic** equation. To get the variable by itself on one side, the square root is used to change x^2 to x. However, the **square root of both sides** must be found. Each quadratic equation has **two solutions**!

Solve for x in the following quadratic equation

$$x^2 + 5 = 21$$
$$x^2 + 5 - 5 = 21 - 5$$

Subtract 5 from both sides

$$x^2 = 16$$

Find the square root of both sides

$$x^2 = (\pm 4)^2$$
$$x = \pm 4$$

Because $4^2 = 16$ and $(-4)^2 = 16$

So $x = -4$ or $x = 4$

3. Find the value of the variable in each of these linear equations:

a $2x = 8$

(÷2) (÷2)

$x = 4$

b $5x = 35$

(÷5) (÷5)

$x = 7$

c $\frac{x}{4} = 5$

(×4) (×4)

$x = 20$

d $\frac{x}{3} = 8$

(×3) (×3)

$x = 24$

e $\frac{3x}{2} = 9$

(×2) (×2)

$3x = 18$

(÷3) (÷3)

$x = 6$

f $\frac{5x}{3} = 5$

(×3) (×3)

$5x = 15$

(÷5) (÷5)

$x = 3$

4. Solve for the variable in each linear equation:

a) $4x + 3 = 23$

(−3) (−3)

$4x = 20$

(÷4) (÷4)

$x = 5$

b) $2a - 5 = 9$

(+5) (+5)

$2a = 14$

(÷2) (÷2)

$a = 7$

c) $5m + 6 = 31$

d) $-4 + 7n = 24$

(+4) (+4)

$7n = 28$

(÷7) (÷7)

$n = 4$

e) $-2k = 10$

(÷−2) (÷−2)

$k = -5$

f) $-m + 4 = 6$ (Hint: $-m = -1m$)

(−4) (−4)

$-m = 2$

(÷−1) (÷−1)

$m = -2$

What happens if the Variable is on BOTH sides of the Equation?

If the variables appear on both sides of the equation, one side needs to be changed to have no variables in it. REMEMBER! Whatever is done to one side must be done to the other side too.

Solve this linear equation:

$$8x - 4 = 3x + 6$$

$3x$ can be subtracted from both sides. Now the right hand side will have no variables.

$$8x - 4 - 3x = 3x + 6 - 3x$$

$$5x - 4 = 6$$

$$5x - 4 + 4 = 6 + 4$$

Add 4 to both sides so that $5x$ is by itself

$$5x = 10$$

Divide both sides by 5 so that x is by itself

$$\frac{5x}{5} = \frac{10}{5}$$

$$x = 2$$

Solve this linear equation:

$$3p - 12 = 16 - 4p$$

$4p$ can be added to both sides. Now the right hand side will have no variables.

$$3p - 12 + 4p = 16 - 4p + 4p$$

$$7p - 12 = 16$$

$$7p - 12 + 12 = 16 + 12$$

Add 12 to both sides so that $7p$ is by itself

$$7p = 28$$

Divide both sides by 7 so that p is by itself

$$\frac{7p}{7} = \frac{28}{7}$$

$$p = 4$$

If the equation has brackets, they must be expanded first. Then just solve the equation the same way as before.

Solve the following linear equation:

Expand both brackets

$$3(x - 3) = 2(x - 2)$$

$$3x - 9 = 2x - 4$$

$$3x - 9 - 2x - 4 - 2x$$

$2x$ can be subtracted from both sides so that the right side will have no variables

$$x - 9 = -4$$

Add 9 to both sides so that x is by itself

$$x - 9 + 9 = -4 + 9$$

$$x = 5$$

1. In these linear equations the variable appears on boths sides. Solve for the missing value:

a) $2u - 10 = 3u$

(−2u) (−2u)

10 −

b) $7x - 18 = 3x + 10$

(−3x) (−3x)

4x − 18 = 10

(+18) (+18)

4x = 28

(÷4) x = 7 (÷4)

c) $3(x + 2) = -3$

3x + 6 = −3

(−6) 3x = −9 (−6)

(÷3) (÷3)

x = −3

d) $4y + 18 = 12 - 2y$

(+2y) (+2y)

6y + 18 = 12

(−18) 6y = −6 (−18)

(÷6) y = −1 (÷6)

e) $10(n - 6) = 2(10 + n)$

10n − 60 = 20 + 2n

(−2n) 8n − 60 = 20 (−2n)

(+60) 8n = 80 (+60)

(÷8) n = 10 (÷8)

f) $6m - 4 = -5(m + 3)$

6m − 4 = −5m − 15

(+5m) (+5m)

11m − 4 = −15

(+4) 11m = −11 (+4)

(÷11) m = −1 (÷11)

g) $8(k - 4) - 5k + 3 = 4$

8k − 32 − 5k + 3 = 4

h) $5(2y - 1) - 6(y - 2) + 3 = 6$

10y − 5 − 6y + 12 + 3 = 6

4y + 10 = 6

(−10) (−10)

4y = −4

(÷4) y = −1 (÷4)

i) $8t - (2t - 18) = -12$

8t − 2t + 18 = −12

6t + 18 = −12

(−18) (−18)

j) $2(a + 3) - 3(a + 4) = -10$

2a + 6 − 3a − 12 = −10

2. Find Ivan's mistake when he tried to solve this equation?

$$3(h + 2) = 2(h + 1) + 5$$

$$3h + 2 = 2h + 2 + 5$$

$$3h + 2 = 2h + 7$$

$$3h + 2 - 2h - 2 = 2h + 7 - 2h - 2$$

$$h = 5$$